

<b>COURSE:</b> Honors Functions	<b>GRADE(S):</b> 11
<b>UNIT 1:</b> Functions and Graphs	<b>TIME FRAME:</b> 14 to 26 days

<p><b>NCTM STANDARDS:</b></p> <p><b>1. NUMBER AND OPERATIONS</b>  A. Understand numbers, ways of representing numbers, relationships among numbers, and number systems  B. Understand meanings of operations and how they relate to one another  C. Compute fluently and make reasonable estimates</p> <p><b>2. ALGEBRA</b>  A. Understand patterns, relations, and functions  B. Represent and analyze mathematical situations and structures using algebraic symbols  C. Use mathematical models to represent and understand quantitative relationships  D. Analyze change in various contexts</p> <p><b>3. GEOMETRY</b>  B. Specify locations and describe spatial relationships using coordinate geometry and other representational systems  C. Apply transformations and use symmetry to analyze mathematical situations  D. Use visualization, spatial reasoning, and geometric modeling to solve problems</p> <p><b>4. MEASUREMENT</b>  A. Understand measurable attributes of objects and the units, systems, and processes of measurement  B. Apply appropriate techniques, tools, and formulas to determine measurements</p> <p><b>6. PROBLEM SOLVING</b>  A. Build new mathematical knowledge through problem solving  B. Solve problems that arise in mathematics and in other contexts  C. Apply and adapt a variety of appropriate strategies to solve problems  D. Monitor and reflect on the process of mathematical problem solving</p> <p><b>7. REASONING AND PROOF</b>  A. Recognize reasoning and proof as fundamental aspects of mathematics  B. Make and investigate mathematical conjectures  C. Develop and evaluate mathematical arguments and proofs  D. Select and use various types of reasoning and methods of proof</p> <p><b>8. COMMUNICATION</b>  A. Organize and consolidate their mathematical thinking through communication  B. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others  C. Analyze and evaluate the mathematical thinking and strategies of others  D. Use the language of mathematics to express mathematical ideas precisely</p> <p><b>9. CONNECTIONS</b>  A. Recognize and use connections among mathematical ideas  B. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole  C. Recognize and apply mathematics in contexts outside of mathematics</p> <p><b>10. REPRESENTATION</b>  A. Create and use representations to organize, record, and communicate mathematical ideas  B. Select, apply, and translate among mathematical representations to solve problems  C. Use representations to model and interpret physical, social, and mathematical phenomena</p>
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PA ELIGIBLE CONTENT:	UNIT OBJECTIVES:
<p><b>M11.A.3.1.1</b></p> <ul style="list-style-type: none"> <li>Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).</li> </ul> <p><b>M11.D.1.1.1</b></p> <ul style="list-style-type: none"> <li>Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.</li> </ul> <p><b>M11.D.1.1.2</b></p> <ul style="list-style-type: none"> <li>Determine if a relation is a function given a set of points or a graph.</li> </ul> <p><b>M11.D.1.1.3</b></p> <ul style="list-style-type: none"> <li>Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).</li> </ul> <p><b>M11.D.2.1.2</b></p> <ul style="list-style-type: none"> <li>Identify or graph functions, linear equations or linear inequalities on a coordinate plane.</li> </ul> <p><b>M11.D.2.1.3</b></p> <ul style="list-style-type: none"> <li>Write, solve and/or apply a linear equation (including problem situations).</li> </ul> <p><b>M11.D.2.1.5</b></p> <ul style="list-style-type: none"> <li>Solve quadratic equations using factoring (integers only – not including completing the square or the Quadratic Formula).</li> </ul> <p><b>M11.D.3.1.1</b></p> <ul style="list-style-type: none"> <li>Identify, describe and/or use constant or varying rates of change.</li> </ul> <p><b>M11.D.3.1.2</b></p> <ul style="list-style-type: none"> <li>Determine how a change in one variable relates to a change in a second variable (e.g., <math>y=4/x</math>, if <math>x</math> doubles, what happens to <math>y</math>?).</li> </ul> <p><b>M11.D.4.1.1</b></p> <ul style="list-style-type: none"> <li>Match the graph of a given function to its table or equation.</li> </ul> <p><b>M11.E.4.2.1</b></p> <ul style="list-style-type: none"> <li>Draw, find and/or write an equation for a line of best fit for a scatter plot.</li> </ul> <p><b>M11.E.4.2.2</b></p> <ul style="list-style-type: none"> <li>Make predictions using the equations or graphs of best-fit lines of scatter plots.</li> </ul>	<ol style="list-style-type: none"> <li>Use algebraic, numerical, and graphical models to solve problems.</li> <li>Analyze the characteristics of the basic functions.</li> <li>Use basic functions to build new functions.</li> <li>Define functions and relations parametrically.</li> <li>Find the inverse of a relation or function.</li> <li>Investigate transformations of functions and parametric relations.</li> <li>Use concepts of functions in real world situations.</li> </ol>

<p><b>ACTIVITIES:</b></p> <ol style="list-style-type: none"> <li>1. <i>Activity: Mathematical Definitions: Precalculus</i></li> <li>2. Represent problems using different models</li> <li>3. Fit curves to data</li> <li>4. Solve equations algebraically</li> <li>5. <i>Activity: Ten Commandments of Mathematics</i></li> <li>6. <i>Activity: Algebra Card Tricks</i></li> <li>7. Investigate grapher failure</li> <li>8. Examine the graphical representations of functions</li> <li>9. Determine the domain and range of a function</li> <li>10. Investigate the continuity of functions</li> <li>11. Identify local extrema</li> <li>12. Find the asymptotes of a function</li> <li>13. Identify and analyze the twelve basic functions</li> <li>14. Add, subtract, multiply, and divide functions</li> <li>15. Find the composition of functions</li> <li>16. Determine the domain of a composition</li> <li>17. Use implicitly defined functions to define relations</li> <li>18. Investigate parametric equations</li> <li>19. <i>Activity: Crashing Airplanes</i></li> <li>20. Find the inverse of a relation or function</li> <li>21. Use the horizontal line test to determine if a relation has an inverse</li> <li>22. Determine whether a function is one-to-one</li> <li>23. Find equations for translations reflections, stretches and shrinks of functions</li> <li>24. Examine combinations of transformations and the consequences of the order in which they are applied</li> </ol> <p><i>Activity: Examining How Mathematics is Used in the Workplace</i>  <i>Activity: The Point of No Return</i>  <i>Lab Activity: It Averages Out in the End</i></p>	<p><b>ASSESSMENTS:</b></p> <ul style="list-style-type: none"> <li>• Observation and questioning</li> <li>• Presentations and discussions</li> <li>• Projects and investigations</li> <li>• Mathematical writing</li> <li>• Homework</li> <li>• Quizzes</li> <li>• Tests</li> </ul> <p><b>REMEDIATION:</b></p> <ul style="list-style-type: none"> <li>• Slope and Y-Intercept 1</li> <li>• X- and Y-Intercepts</li> <li>• Slope and Y-Intercept 2</li> <li>• Graphing Quadratics 1, 2, and 3</li> <li>• Quick Graphs of Quadratic Equations</li> <li>• Manipulating Powers (2 pages)</li> <li>• Evaluating Rational Exponents</li> <li>• Simplifying Radicals</li> <li>• Domain and Range #1</li> <li>• Domain and Range #2</li> <li>• Sum, Difference, and Product</li> <li>• Conjugate and Quotient</li> </ul> <p><b>ENRICHMENT:</b></p> <ul style="list-style-type: none"> <li>• Red-Haired Older Son</li> <li>• Project: Collaborative Investigation— Babylonian Square Roots</li> </ul> <p><b>DIFFERENTIATION</b>  <a href="http://www.algebrahelp.com">www.algebrahelp.com</a>  <a href="http://www.coolmath.com">www.coolmath.com</a>  <a href="http://www.mathleague.com">www.mathleague.com</a>  <a href="http://www.interactmath.com">www.interactmath.com</a></p> <p><b>RESOURCES:</b>  Precalculus: Graphing, Numerical, Algebraic, 7th ed.,  Demana et. al., © 2007</p>
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<b>COURSE: Honors Functions</b>	<b>GRADE(S): 11</b>
<b>UNIT 2: Polynomial, Power, and Rational Functions</b>	
<b>TIME FRAME: 12 to 14 days</b>	

**NCTM STANDARDS:**

**1. NUMBER AND OPERATIONS**

- A. Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- B. Understand meanings of operations and how they relate to one another
- C. Compute fluently and make reasonable estimates

**2. ALGEBRA**

- A. Understand patterns, relations, and functions
- B. Represent and analyze mathematical situations and structures using algebraic symbols
- C. Use mathematical models to represent and understand quantitative relationships
- D. Analyze change in various contexts

**3. GEOMETRY**

- C. Apply transformations and use symmetry to analyze mathematical situations
- D. Use visualization, spatial reasoning, and geometric modeling to solve problems

**4. MEASUREMENT**

- A. Understand measurable attributes of objects and the units, systems, and processes of measurement
- B. Apply appropriate techniques, tools, and formulas to determine measurements

**5. DATA ANALYSIS AND PROBABILITY**

- C. Develop and evaluate inferences and predictions that are based on data

**6. PROBLEM SOLVING**

- A. Build new mathematical knowledge through problem solving
- B. Solve problems that arise in mathematics and in other contexts
- C. Apply and adapt a variety of appropriate strategies to solve problems
- D. Monitor and reflect on the process of mathematical problem solving

**7. REASONING AND PROOF**

- A. Recognize reasoning and proof as fundamental aspects of mathematics
- B. Make and investigate mathematical conjectures
- C. Develop and evaluate mathematical arguments and proofs
- D. Select and use various types of reasoning and methods of proof

**8. COMMUNICATION**

- A. Organize and consolidate their mathematical thinking through communication
- B. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- C. Analyze and evaluate the mathematical thinking and strategies of others
- D. Use the language of mathematics to express mathematical ideas precisely

**9. CONNECTIONS**

- A. Recognize and use connections among mathematical ideas
- B. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- C. Recognize and apply mathematics in contexts outside of mathematics

**10. REPRESENTATION**

- A. Create and use representations to organize, record, and communicate mathematical ideas
- B. Select, apply, and translate among mathematical representations to solve problems
- C. Use representations to model and interpret physical, social, and mathematical phenomena

**PA MATH ASSESSMENT ANCHORS:****M11.A.2.2.1**

- Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).

**M11.A.2.2.2**

- Simplify/evaluate expressions involving multiplying with exponents (e.g.  $x^6 * x^7 = x^{13}$ ), powers of powers (e.g.,  $(x^6)^7 = x^{42}$ ) and powers of products  $(2x^2)^3 = 8x^6$  (positive exponents only).

**M11.A.3.1.1**

- Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).

**M11.D.1.1.1**

- Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.

**M11.D.1.1.2**

- Determine if a relation is a function given a set of points or a graph.

**M11.D.1.1.3**

- Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

**M11.D.2.1.2**

- Identify or graph functions, linear equations or linear inequalities on a coordinate plane.

**M11.D.2.1.3**

- Write, solve and/or apply a linear equation (including problem situations).

**M11.D.2.1.5**

- Solve quadratic equations using factoring (integers only – not including completing the square or the Quadratic Formula).

**M11.D.2.2.1**

- Add, subtract and/or multiply polynomial expressions (express answers in simplest form – nothing larger than a binomial multiplied by a trinomial).

**UNIT OBJECTIVES:**

1. Graph polynomial functions.
2. Investigate power functions.
3. Predict end behavior and determine the real zeros of polynomial functions.
4. Determine rational zeros using the rational roots theorem, the factor theorem and synthetic division.
5. Investigate complex zeros.
6. Analyze graphs of rational functions.
7. Use graphical and algebraic techniques to solve equations and inequalities.

<p><b>M11.D.2.2.2</b></p> <ul style="list-style-type: none"> <li>Factor algebraic expressions, including difference of squares and trinomials (trinomials limited to the form <math>ax^2+bx+c</math> where <math>a</math> is not equal to 0).</li> </ul> <p><b>M11.D.2.2.3</b></p> <ul style="list-style-type: none"> <li>Simplify algebraic fractions.</li> </ul> <p><b>M11.D.3.1.1</b></p> <ul style="list-style-type: none"> <li>Identify, describe and/or use constant or varying rates of change.</li> </ul> <p><b>M11.D.3.1.2</b></p> <ul style="list-style-type: none"> <li>Determine how a change in one variable relates to a change in a second variable (e.g., <math>y=4/x</math>, if <math>x</math> doubles, what happens to <math>y</math>?).</li> </ul> <p><b>M11.D.4.1.1</b></p> <ul style="list-style-type: none"> <li>Match the graph of a given function to its table or equation.</li> </ul> <p><b>M11.E.4.2.1</b></p> <ul style="list-style-type: none"> <li>Draw, find and/or write an equation for a line of best fit for a scatter plot.</li> </ul> <p><b>M11.E.4.2.2</b></p> <ul style="list-style-type: none"> <li>Make predictions using the equations or graphs of best-fit lines of scatter plots.</li> </ul>	
<p><b>ACTIVITIES:</b></p> <ol style="list-style-type: none"> <li>Graph polynomial functions of various degrees and investigate the shapes of the graphs</li> <li>Find the rate of change of a function</li> <li>Use regression models to solve problems</li> <li>Determine the vertex and axis of symmetry for a quadratic function</li> <li>Determine local maximum and minimum values</li> <li>Analyze the characteristics of power functions</li> <li>Use power functions to model real life problems</li> <li>Investigate the end behavior of polynomial functions of even and odd degree</li> <li>Find the end behavior model</li> <li>Determine the zeros of a polynomial function</li> <li>Establish the multiplicity of the zeros of a polynomial function</li> </ol>	<p><b>ASSESSMENTS:</b></p> <ul style="list-style-type: none"> <li>Observation and questioning</li> <li>Presentations and discussions</li> <li>Projects and investigations</li> <li>Mathematical writing</li> <li>Homework</li> <li>Quizzes</li> <li>Tests</li> </ul> <p><b>REMEDICATION:</b></p> <ul style="list-style-type: none"> <li>Synthetic Substitution</li> <li>Synthetic Substitution (cont.)</li> <li>The Remainder Theorem</li> <li>The Factor Theorem</li> <li>Dividing Polynomials</li> <li>Synthetic Division</li> </ul> <p><b>ENRICHMENT:</b></p> <ul style="list-style-type: none"> <li>Einstein's Problem</li> </ul>

12. Explore the intermediate value property algebraically and graphically
13. Explore the division algorithm for polynomials
14. Use the remainder and factor theorem to test for zeros
15. Use synthetic division as an aid to test for rational zeros
16. Test for rational zeros using the rational roots theorem
17. Determine the upper and lower bounds for real zeros
18. Model real-world situations using polynomial functions
19. Perform operations with complex numbers
20. Determine complex zeros
21. Investigate polynomial functions of even and odd degree and their possible number of real and complex zeros
22. Find the domain of a rational function
23. Determine the asymptotes of a rational function
24. Graph rational functions
25. Solve rational equations
26. Determine when a rational equations has an extraneous solution
27. Use rational functions to solve real world problems

*Activity: Designing a Juice Can*

Solve polynomial and rational inequalities algebraically and graphically  
Use a sign chart to solve inequalities

**DIFFERENTIATION:**

<http://www.sosmath.com/index.html>  
[www.algebrahelp.com](http://www.algebrahelp.com)  
[www.coolmath.com](http://www.coolmath.com)  
[www.mathleague.com](http://www.mathleague.com)  
[www.interactmath.com](http://www.interactmath.com)

**RESOURCES:**

Precalculus: Graphing, Numerical, Algebraic, 7th ed.,  
Demana et. al., © 2007

<b>COURSE: Honors Functions</b>	<b>GRADE(S): 11</b>
<b>UNIT 3: Exponential, Logistic, and Logarithmic Functions</b>	<b>TIME FRAME: 12 to 14 Days</b>

**NCTM STANDARDS:**

**1. NUMBER AND OPERATIONS**

- A. Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- B. Understand meanings of operations and how they relate to one another
- C. Compute fluently and make reasonable estimates

**2. ALGEBRA**

- A. Understand patterns, relations, and functions
- B. Represent and analyze mathematical situations and structures using algebraic symbols
- C. Use mathematical models to represent and understand quantitative relationships
- D. Analyze change in various contexts

**3. GEOMETRY**

- C. Apply transformations and use symmetry to analyze mathematical situations
- D. Use visualization, spatial reasoning, and geometric modeling to solve problems

**4. MEASUREMENT**

- A. Understand measurable attributes of objects and the units, systems, and processes of measurement
- B. Apply appropriate techniques, tools, and formulas to determine measurements

**5. DATA ANALYSIS AND PROBABILITY**

- C. Develop and evaluate inferences and predictions that are based on data

**6. PROBLEM SOLVING**

- A. Build new mathematical knowledge through problem solving
- B. Solve problems that arise in mathematics and in other contexts
- C. Apply and adapt a variety of appropriate strategies to solve problems
- D. Monitor and reflect on the process of mathematical problem solving

**7. REASONING AND PROOF**

- A. Recognize reasoning and proof as fundamental aspects of mathematics
- B. Make and investigate mathematical conjectures
- C. Develop and evaluate mathematical arguments and proofs
- D. Select and use various types of reasoning and methods of proof

**8. COMMUNICATION**

- A. Organize and consolidate their mathematical thinking through communication
- B. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- C. Analyze and evaluate the mathematical thinking and strategies of others
- D. Use the language of mathematics to express mathematical ideas precisely

**9. CONNECTIONS**

- A. Recognize and use connections among mathematical ideas
- B. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- C. Recognize and apply mathematics in contexts outside of mathematics

**10. REPRESENTATION**

- A. Create and use representations to organize, record, and communicate mathematical ideas
- B. Select, apply, and translate among mathematical representations to solve problems
- C. Use representations to model and interpret physical, social, and mathematical phenomena



**PA MATH ASSESSMENT ANCHORS:****M11.A.2.2.1**

- Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).

**M11.A.2.2.2**

- Simplify/evaluate expressions involving multiplying with exponents (e.g.  $x^6 * x^7 = x^{13}$ ), powers of powers (e.g.,  $(x^6)^7 = x^{42}$ ) and powers of products  $(2x^2)^3 = 8x^6$  (positive exponents only).

**M11.A.3.1.1**

- Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).

**M11.D.1.1.1**

- Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.

**M11.D.1.1.2**

- Determine if a relation is a function given a set of points or a graph.

**M11.D.1.1.3**

- Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

**M11.D.2.1.2**

- Identify or graph functions, linear equations or linear inequalities on a coordinate plane.

**M11.D.2.1.3**

- Write, solve and/or apply a linear equation (including problem situations).

**M11.D.2.1.5**

- Solve quadratic equations using factoring (integers only – not including completing the square or the Quadratic Formula).

**M11.D.2.2.1**

- Add, subtract and/or multiply polynomial expressions (express answers in simplest form – nothing larger than a binomial multiplied by a trinomial).

**UNIT OBJECTIVES:**

1. Evaluate exponential expressions and graph exponential functions.
2. Use exponential growth and decay to model real-life problems.
3. Evaluate and graph common and natural Logarithms.
4. Apply the properties of logarithms to solve exponential and logarithmic equations algebraically.
5. Solve a variety of application problems involving logarithms and use exponential functions to solve business and finance applications.

<p><b>M11.D.2.2.2</b></p> <ul style="list-style-type: none"> <li>Factor algebraic expressions, including difference of squares and trinomials (trinomials limited to the form <math>ax^2+bx+c</math> where <math>a</math> is not equal to 0).</li> </ul> <p><b>M11.D.2.2.3</b></p> <ul style="list-style-type: none"> <li>Simplify algebraic fractions.</li> </ul> <p><b>M11.D.3.1.1</b></p> <ul style="list-style-type: none"> <li>Identify, describe and/or use constant or varying rates of change.</li> </ul> <p><b>M11.D.3.1.2</b></p> <ul style="list-style-type: none"> <li>Determine how a change in one variable relates to a change in a second variable (e.g., <math>y=4/x</math>, if <math>x</math> doubles, what happens to <math>y</math>?).</li> </ul> <p><b>M11.D.4.1.1</b></p> <ul style="list-style-type: none"> <li>Match the graph of a given function to its table or equation.</li> </ul> <p><b>M11.E.4.2.1</b></p> <ul style="list-style-type: none"> <li>Draw, find and/or write an equation for a line of best fit for a scatter plot.</li> </ul> <p><b>M11.E.4.2.2</b></p> <ul style="list-style-type: none"> <li>Make predictions using the equations or graphs of best-fit lines of scatter plots.</li> </ul>	
<p><b>ACTIVITIES:</b></p> <ol style="list-style-type: none"> <li>Graph exponential and logarithmic functions</li> <li>Perform transformations of the graphs of exponential functions</li> <li>Investigate the natural exponential functions</li> <li>Apply exponential functions to the real-world situations of growth and decay</li> </ol> <p><i>Activity: The M&amp;M Function</i>  <i>Activity: Carbon Dating</i></p> <ol style="list-style-type: none"> <li>Apply logistic growth functions to real-world situations</li> <li>Investigate the inverse of the exponential function</li> <li>Graph the logarithmic function</li> <li>Change functions from exponential to logarithmic form</li> </ol> <p><i>Activity: Logarithmic Equations</i></p> <ol style="list-style-type: none"> <li>Evaluate logarithmic expressions</li> <li>Use the properties of logarithms to evaluate expressions</li> <li>Perform transformations on the graph of the logarithmic function</li> </ol>	<p><b>ASSESSMENTS:</b></p> <ul style="list-style-type: none"> <li>Observation and questioning</li> <li>Presentations and discussions</li> <li>Projects and investigations</li> <li>Mathematical writing</li> <li>Homework</li> <li>Quizzes</li> <li>Tests</li> </ul> <p><b>REMEDIATION:</b></p> <ul style="list-style-type: none"> <li>The Inverse of a Function</li> <li>Graphing the Inverse of a Function</li> <li>Logarithm Combination Rules</li> <li>Solving Exponential Equations with Logs #1</li> <li>Solving Exponential Equations with Logs (cont.) #2</li> <li>Compound Interest #1</li> <li>Compound Interest #2</li> <li>Simplifying Logarithms</li> <li>Simplifying and Solving Logarithms</li> <li>Continuous Growth and Radioactive Decay</li> </ul>

12. Use the change of base theorem to evaluate logs of different bases
13. Use exponential and logarithmic properties to solve logarithmic equations
14. Explore financial applications of exponential functions

*Activity: Compound Interest*

*Activity: Modeling with Exponential and Logarithmic Equations*

*Functions: Guess the Power*

*Activity: Bank Account*

*Activity: Why Does the Rule of 72 Work?*

**DIFFERENTIATION:**

<http://www.sosmath.com/index.html>

[www.algebrahelp.com](http://www.algebrahelp.com)

[www.coolmath.com](http://www.coolmath.com)

[www.mathleague.com](http://www.mathleague.com)

[www.interactmath.com](http://www.interactmath.com)

**ENRICHMENT:**

- *Activity: Are Colleges Still Affordable?*
- Hazards of Heavy Metal: An Investigation Using Exponential Models
- Functions
- Project: A Graphical Approach to Compound Interest
- Logarithmic Scale

**RESOURCES:**

Precalculus: Graphing, Numerical, Algebraic, 7th ed.,

Demana et. al., © 2007

<b>COURSE:</b> Honors Functions	<b>GRADE(S):</b> 11
<b>UNIT 4:</b> Trigonometric Functions	<b>TIME FRAME:</b> 10 to 12 Days

**NCTM STANDARDS:**

**1. NUMBER AND OPERATIONS**

- A. Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- B. Understand meanings of operations and how they relate to one another
- C. Compute fluently and make reasonable estimates

**2. ALGEBRA**

- A. Understand patterns, relations, and functions
- B. Represent and analyze mathematical situations and structures using algebraic symbols
- C. Use mathematical models to represent and understand quantitative relationships
- D. Analyze change in various contexts

**3. GEOMETRY**

- C. Apply transformations and use symmetry to analyze mathematical situations
- D. Use visualization, spatial reasoning, and geometric modeling to solve problems

**4. MEASUREMENT**

- A. Understand measurable attributes of objects and the units, systems, and processes of measurement
- B. Apply appropriate techniques, tools, and formulas to determine measurements

**6. PROBLEM SOLVING**

- A. Build new mathematical knowledge through problem solving
- B. Solve problems that arise in mathematics and in other contexts
- C. Apply and adapt a variety of appropriate strategies to solve problems
- D. Monitor and reflect on the process of mathematical problem solving

**7. REASONING AND PROOF**

- A. Recognize reasoning and proof as fundamental aspects of mathematics
- B. Make and investigate mathematical conjectures
- C. Develop and evaluate mathematical arguments and proofs
- D. Select and use various types of reasoning and methods of proof

**8. COMMUNICATION**

- A. Organize and consolidate their mathematical thinking through communication
- B. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- C. Analyze and evaluate the mathematical thinking and strategies of others
- D. Use the language of mathematics to express mathematical ideas precisely

**9. CONNECTIONS**

- A. Recognize and use connections among mathematical ideas
- B. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- C. Recognize and apply mathematics in contexts outside of mathematics

**10. REPRESENTATION**

- A. Create and use representations to organize, record, and communicate mathematical ideas
- B. Select, apply, and translate among mathematical representations to solve problems
- C. Use representations to model and interpret physical, social, and mathematical phenomena

**PA MATH ASSESSMENT ANCHORS:**

**M11.A.2.2.2**

- Simplify/evaluate expressions involving multiplying with exponents (e.g.  $x^6 * x^7 = x^{13}$ ), powers of powers (e.g.,  $(x^6)^7=x^{42}$ ) and powers of products  $(2x^2)^3=8x^6$  (positive exponents only).

**M11.A.3.1.1**

- Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).

**M11.D.1.1.1**

- Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.

**M11.D.1.1.2**

- Determine if a relation is a function given a set of points or a graph.

**M11.D.1.1.3**

- Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).

**M11.D.2.1.2**

- Identify or graph functions, linear equations or linear inequalities on a coordinate plane.

**M11.D.2.2.3**

- Simplify algebraic fractions.

**M11.D.3.1.1**

- Identify, describe and/or use constant or varying rates of change.

**M11.D.3.1.2**

- Determine how a change in one variable relates to a change in a second variable (e.g.,  $y=4/x$ , if  $x$  doubles, what happens to  $y$ ?).

**M11.D.4.1.1**

- Match the graph of a given function to its table or equation.

**UNIT OBJECTIVES:**

1. Determine and use central angle measure in radians and degrees.
2. Determine the trigonometric functions of an acute angle with respect to a right triangle.
3. Investigate the trigonometric functions with respect to the unit circle.
4. Investigate the graphs of the six trigonometric functions.
5. Graph composite functions involving trigonometric functions.
6. Relate the concept of inverse functions to trigonometric functions.
7. Apply concepts of trigonometry to real world situations.

**ACTIVITIES:**

1. Convert between degrees and radians
2. Define degree and radian measure of an angle
3. Find circular arc length in degrees and radians

**ASSESSMENTS:**

- Observation and questioning Presentations and discussions
- Projects and investigations
- Mathematical writing
- Homework

4. Use angular and linear speed to solve practical problems
5. Define the six trigonometric functions of an acute Angle

*Activity: Some Mnemonics to Remember Your Trig Ratios*

6. Use the special right triangles to evaluate the value of the trigonometric functions for 30, 45, and 60 degrees

*Activity: Trig Cut Ups*

7. Use one trigonometric ratio to find the remaining ones
8. Use a calculator to find the values of trigonometric functions
9. Use trigonometric functions to find the sides of right triangles
10. Explore co-terminal angles
11. Investigate first quadrant trigonometry
12. Use reference triangles to evaluate the trigonometric functions of any angle
13. Determine the trigonometric functions for the quadrantal angles
14. Explore the unit circle and the wrapping function
15. Find trigonometric functions of real numbers
16. Investigate the concept of periodicity
17. Investigate the 16-point unit circle

*Activity: Radian, the Snowman*

*Activity: Radian Walk*

18. Investigate the characteristics of the sine and cosine Functions

*Activity: Sine Cosine Game*

19. Explore the transformations of the sine function
20. Determine the amplitude, period, frequency and phase shift of a sinusoid
21. Investigate the characteristics of the tangent
22. Cotangent, secant, and cosecant graphs
23. Investigate the result of combining trigonometric and algebraic functions
24. Determine when a composite function is periodic
25. Explore sums and differences of sinusoids
26. Determine whether or not a function is a sinusoid
27. Investigate the domain and range of the inverse functions
28. Evaluate inverse functions with and without a calculator
29. Evaluate compositions of trigonometric

- Quizzes
- Tests

#### REMEDIATION:

- Manipulating Special Right Triangles
- Trigonometric Ratios
- Evaluating Trigonometric Functions
- Applying Trigonometric Ratios
- Using Trigonometric Ratios to Find Angles
- Trigonometric Ratios
- Angles Greater Than 360 Degrees
- Converting Angle Measurements
- Manipulating Properties of Sine and Cosine
- Graphing Sine and Cosine Functions
- Graphing the Sine and Cosine Functions (cont.)
- Graphing  $y = a \sin x$  or  $y = a \cos x$
- Graphing  $y = c + \sin x$  or  $y = c + \cos x$
- Graphing  $y = \sin bx$  or  $y = \cos bx$

#### DIFFERENTIATION

<http://www.sosmath.com/index.html>

[www.coolmath.com](http://www.coolmath.com)

[www.mathleague.com](http://www.mathleague.com)

[www.interactmath.com](http://www.interactmath.com)

<http://www.themathpage.com/aTrig/trigonometry.htm>

#### ENRICHMENT:

- Project: Fitting a Model to Data

#### RESOURCES:

Precalculus: Graphing, Numerical, Algebraic, 7th ed.,

Demana et. al., © 2007

<p>and inverse trigonometric functions</p> <ol style="list-style-type: none"><li>30. Solve right triangles</li><li>31. Apply right triangle trigonometry to real-world situations</li><li>32. Solve trigonometric equations and inequalities algebraically and graphically</li><li>33. Use trigonometric functions to determine the angle between lines</li><li>34. Use angle of depression and the angle of elevation in application problems</li><li>35. Solve trigonometric equations and inequalities algebraically and graphically</li></ol>	
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<b>COURSE:</b> Honors Functions	<b>GRADE(S):</b> 11 <sup>th</sup>
<b>UNIT 5: Analytic Trigonometry</b>	<b>TIME FRAME:</b> 12 to 14 days

<p><b>NATIONAL STANDARDS: NCTM Standards</b></p> <p><b>1. NUMBER AND OPERATIONS</b></p> <p>A. Understand numbers, ways of representing numbers, relationships among numbers, and number systems</p> <p>B. Understand meanings of operations and how they relate to one another</p> <p>C. Compute fluently and make reasonable estimates</p> <p><b>2. ALGEBRA</b></p> <p>A. Understand patterns, relations, and functions</p> <p>B. Represent and analyze mathematical situations and structures using algebraic symbols</p> <p>C. Use mathematical models to represent and understand quantitative relationships</p> <p>D. Analyze change in various contexts</p> <p><b>3. GEOMETRY</b></p> <p>A. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships</p> <p>B. Specify locations and describe spatial relationships using coordinate geometry and other representational systems</p> <p>C. Apply transformations and use symmetry to analyze mathematical situations</p> <p>D. Use visualization, spatial reasoning, and geometric modeling to solve problems</p> <p><b>4. MEASUREMENT</b></p> <p>A. Understand measurable attributes of objects and the units, systems, and processes of measurement</p> <p>B. Apply appropriate techniques, tools, and formulas to determine measurements</p> <p><b>6. PROBLEM SOLVING</b></p> <p>A. Build new mathematical knowledge through problem solving</p> <p>B. Solve problems that arise in mathematics and in other contexts</p> <p>C. Apply and adapt a variety of appropriate strategies to solve problems</p> <p>D. Monitor and reflect on the process of mathematical problem solving</p> <p><b>7. REASONING AND PROOF</b></p> <p>A. Recognize reasoning and proof as fundamental aspects of mathematics</p> <p>B. Make and investigate mathematical conjectures</p> <p>C. Develop and evaluate mathematical arguments and proofs</p> <p><b>8. COMMUNICATION</b></p> <p>A. Organize and consolidate their mathematical thinking through communication</p> <p>B. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others</p> <p>C. Analyze and evaluate the mathematical thinking and strategies of others</p> <p>D. Use the language of mathematics to express mathematical ideas precisely</p> <p><b>9. CONNECTIONS</b></p> <p>A. Recognize and use connections among mathematical ideas</p> <p>B. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole</p> <p>C. Recognize and apply mathematics in contexts outside of mathematics</p> <p><b>10. REPRESENTATION</b></p> <p>A. Create and use representations to organize, record, and communicate mathematical ideas</p> <p>B. Select, apply, and translate among mathematical representations to solve problems</p> <p>C. Use representations to model and interpret physical, social, and mathematical phenomena</p>
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PA MATH ASSESSMENT ANCHORS:	UNIT OBJECTIVES:
<p><b>M11.A.2.2.2</b></p> <ul style="list-style-type: none"> <li>Simplify/evaluate expressions involving multiplying with exponents (e.g. <math>x^6 * x^7 = x^{13}</math>), powers of powers (e.g., <math>(x^6)^7 = x^{42}</math>) and powers of products <math>(2x^2)^3 = 8x^6</math> (positive exponents only).</li> </ul> <p><b>M11.A.3.1.1</b></p> <ul style="list-style-type: none"> <li>Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used).</li> </ul> <p><b>M11.D.1.1.1</b></p> <ul style="list-style-type: none"> <li>Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.</li> </ul> <p><b>M11.D.1.1.2</b></p> <ul style="list-style-type: none"> <li>Determine if a relation is a function given a set of points or a graph.</li> </ul> <p><b>M11.D.1.1.3</b></p> <ul style="list-style-type: none"> <li>Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).</li> </ul> <p><b>M11.D.2.1.2</b></p> <ul style="list-style-type: none"> <li>Identify or graph functions, linear equations or linear inequalities on a coordinate plane.</li> </ul> <p><b>M11.D.2.2.3</b></p> <ul style="list-style-type: none"> <li>Simplify algebraic fractions.</li> </ul> <p><b>M11.D.3.1.1</b></p> <ul style="list-style-type: none"> <li>Identify, describe and/or use constant or varying rates of change.</li> </ul> <p><b>M11.D.3.1.2</b></p> <ul style="list-style-type: none"> <li>Determine how a change in one variable relates to a change in a second variable (e.g., <math>y=4/x</math>, if <math>x</math> doubles, what happens to <math>y</math>?).</li> </ul> <p><b>M11.D.4.1.1</b></p> <ul style="list-style-type: none"> <li>Match the graph of a given function to its table or equation.</li> </ul>	<p><b>UNIT OBJECTIVES:</b></p> <ol style="list-style-type: none"> <li>Use the fundamental trigonometric identities to simplify trigonometric expressions and solve trigonometric equations.</li> <li>Prove a variety of trigonometric identities.</li> <li>Understand and use the laws of sines and cosines.</li> <li>Examine the area of a triangle.</li> </ol>

<p><b>ACTIVITIES:</b></p> <ol style="list-style-type: none"> <li>1. Investigate the basic, Pythagorean, co-function, and odd-even identities</li> </ol> <p><i>Activity: Trigonometry Triangles</i></p> <ol style="list-style-type: none"> <li>2. Simplify trigonometric expressions</li> <li>3. Solve trigonometric equations</li> <li>4. Verify trigonometric identities graphically</li> <li>5. Prove a variety of trigonometric identities analytically</li> </ol> <p><i>Activity: Trig Cut Up (2 versions)</i></p> <ol style="list-style-type: none"> <li>6. Disprove non-identities</li> <li>7. Apply the cosine, sine, and tangent sum and difference identities</li> <li>8. Apply the double-angle and half-angle identities</li> </ol> <p><i>Activity: Trigonometric Sum and Difference Identities</i></p> <p><i>Activity: I Have ... Who Has ... Cards</i></p> <p><i>Activity: Trig Card Game</i></p> <p><i>Activity: Trig Dot-to-Dot</i></p> <p><i>Activity: Magic Square</i></p> <ol style="list-style-type: none"> <li>9. Use the laws of sines and cosines to solve right Triangles</li> <li>10. Apply the laws of sines and cosines to real-life situations</li> <li>11. Develop the formula for the area of a triangle in terms of the sine function</li> <li>12. Find the area of a triangle using Heron's formula</li> <li>13. Apply the triangle area formulas to real world problems</li> </ol>	<p><b>ASSESSMENTS:</b></p> <ul style="list-style-type: none"> <li>• Observation and questioning</li> <li>• Presentations and discussions</li> <li>• Projects and investigations</li> <li>• Mathematical writing</li> <li>• Homework</li> <li>• Quizzes</li> <li>• Tests</li> </ul> <p><b>REMEDIATION:</b></p> <ul style="list-style-type: none"> <li>• Law of Sines</li> <li>• Law of Cosines</li> <li>• The Law of Sines and the Law of Cosines</li> <li>• Trig I.D. Problems (2 pages)</li> <li>• Mixed Problems (3 pages)</li> </ul> <p><b>ENRICHMENT:</b></p> <ul style="list-style-type: none"> <li>• The Science and Math Connection</li> <li>• Folding Leg of Card Table</li> <li>• Checkpoint</li> <li>• Modeling the Illumination of the Moon</li> <li>• Modeling the Motion of a Pendulum</li> <li>• Evaluating Modeling Solutions</li> </ul> <p><b>DIFFERENTIATION</b></p> <p><a href="http://www.coolmath.com">www.coolmath.com</a></p> <p><a href="http://www.mathleague.com">www.mathleague.com</a></p> <p><a href="http://www.interactmath.com">www.interactmath.com</a></p> <p><a href="http://www.themathpage.com/aTrig/trigonometry.htm">http://www.themathpage.com/aTrig/trigonometry.htm</a></p> <p><b>RESOURCES:</b></p> <p>Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. al., © 2007</p>
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COURSE: Honors Functions	GRADE(S): 11th
UNIT 6: Vectors, Parametric Equations	TIME FRAME: 7 to 9 days

**NATIONAL STANDARDS: NCTM Standards**

**1. NUMBER AND OPERATIONS**

- A. Understand numbers, ways of representing numbers, relationships among numbers, and number systems
- B. Understand meanings of operations and how they relate to one another
- C. Compute fluently and make reasonable estimates

**2. ALGEBRA**

- A. Understand patterns, relations, and functions
- B. Represent and analyze mathematical situations and structures using algebraic symbols
- C. Use mathematical models to represent and understand quantitative relationships
- D. Analyze change in various contexts

**3. GEOMETRY**

- A. Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- B. Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- C. Apply transformations and use symmetry to analyze mathematical situations
- D. Use visualization, spatial reasoning, and geometric modeling to solve problems

**4. MEASUREMENT**

- A. Understand measurable attributes of objects and the units, systems, and processes of measurement
- B. Apply appropriate techniques, tools, and formulas to determine measurements

**6. PROBLEM SOLVING**

- A. Build new mathematical knowledge through problem solving
- B. Solve problems that arise in mathematics and in other contexts
- C. Apply and adapt a variety of appropriate strategies to solve problems
- D. Monitor and reflect on the process of mathematical problem solving

**7. REASONING AND PROOF**

- A. Recognize reasoning and proof as fundamental aspects of mathematics
- B. Make and investigate mathematical conjectures
- C. Develop and evaluate mathematical arguments and proofs

**8. COMMUNICATION**

- A. Organize and consolidate their mathematical thinking through communication
- B. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- C. Analyze and evaluate the mathematical thinking and strategies of others
- D. Use the language of mathematics to express mathematical ideas precisely

**9. CONNECTIONS**

- A. Recognize and use connections among mathematical ideas
- B. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- C. Recognize and apply mathematics in contexts outside of mathematics

**10. REPRESENTATION**

- A. Create and use representations to organize, record, and communicate mathematical ideas
- B. Select, apply, and translate among mathematical representations to solve problems
- C. Use representations to model and interpret physical, social, and mathematical phenomena

<p><b>PA MATH ASSESSMENT ANCHORS:</b></p> <p><b>M11.A.2.2.1</b></p> <ul style="list-style-type: none"> <li>Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).</li> </ul> <p><b>M11.A.2.2.2</b></p> <ul style="list-style-type: none"> <li>Simplify/evaluate expressions involving multiplying with exponents (e.g. <math>x^6 * x^7 = x^{13}</math>), powers of powers (e.g., <math>(x^6)^7 = x^{42}</math>) and powers of products <math>(2x^2)^3 = 8x^6</math> (positive exponents only).</li> </ul> <p><b>M11.A.3.1.1</b></p> <ul style="list-style-type: none"> <li>Simplify/evaluate expressions using the order of operations to solve problems (any rational numbers may be used)..</li> </ul> <p><b>M11.D.1.1.3</b></p> <ul style="list-style-type: none"> <li>Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).</li> </ul> <p><b>M11.D.2.1.2</b></p> <ul style="list-style-type: none"> <li>Identify or graph functions, linear equations or linear inequalities on a coordinate plane.</li> </ul> <p><b>M11.D.2.1.3</b></p> <ul style="list-style-type: none"> <li>Write, solve and/or apply a linear equation (including problem situations).</li> </ul> <p><b>M11.D.2.1.5</b></p> <ul style="list-style-type: none"> <li>Solve quadratic equations using factoring (integers only – not including completing the square or the Quadratic Formula).</li> </ul> <p><b>M11.D.2.2.2</b></p> <ul style="list-style-type: none"> <li>Factor algebraic expressions, including difference of squares and trinomials (trinomials limited to the form <math>ax^2+bx+c</math> where <math>a</math> is not equal to 0)</li> </ul> <p><b>M11.D.2.2.3</b></p> <ul style="list-style-type: none"> <li>Simplify algebraic fractions.</li> </ul> <p><b>M11.D.3.1.1</b></p> <ul style="list-style-type: none"> <li>Identify, describe and/or use constant or varying rates of change.</li> </ul> <p><b>M11.D.3.1.2</b></p> <ul style="list-style-type: none"> <li>Determine how a change in one variable relates to a change in a second variable (e.g., <math>y=4/x</math>, if <math>x</math> doubles, what happens to <math>y</math>?).</li> </ul>	<p><b>UNIT OBJECTIVES:</b></p> <ol style="list-style-type: none"> <li>Apply the arithmetic of vectors and use vectors to solve real-world problems.</li> <li>Define parametric equations, graph curves parametrically, and solve application problems using parametric equations.</li> </ol>
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<p><b>M11.D.4.1.1</b></p> <ul style="list-style-type: none"> <li>• Match the graph of a given function to its table or equation.</li> </ul>	
<p><b>ACTIVITIES:</b></p> <ol style="list-style-type: none"> <li>1. Classify quantities as either vector or scalar</li> <li>2. Use terminology associated with vectors</li> <li>3. Determine whether or not two vectors are equal</li> <li>4. Find the component form of a vector</li> <li>5. Perform vector addition and scalar multiplication</li> </ol> <p><i>Activity: Adding Vectors Graphically</i></p> <ol style="list-style-type: none"> <li>6. Find the direction angle of a vector</li> <li>7. Use vectors to represent quantities such as force and velocity</li> <li>8. Calculate dot products and find the length of vectors</li> <li>9. Find the angle between vectors</li> <li>10. Apply vectors to problems involving force and work</li> <li>11. Graph parametric equations</li> <li>12. Eliminate the parameter to obtain a rectangular equation in x and y</li> <li>13. Use a grapher to simulate motion</li> </ol>	<p><b>ASSESSMENTS:</b></p> <ul style="list-style-type: none"> <li>• Observation and questioning</li> <li>• Presentations and discussions</li> <li>• Projects and investigations</li> <li>• Mathematical writing</li> <li>• Homework</li> <li>• Quizzes</li> <li>• Tests</li> </ul> <p><b>REMEDIATION:</b></p> <ul style="list-style-type: none"> <li>• Describing Vectors</li> <li>• Adding Vectors Graphically</li> <li>• Vector Addition and Scalar Multiplication</li> <li>• Resolving Vectors</li> <li>• Describing Vectors from x and y Components</li> <li>• Adding Vectors Algebraically</li> <li>• Vector Dot Product</li> <li>• The Angle Between Two Vectors</li> </ul> <p><b>ENRICHMENT:</b></p> <ul style="list-style-type: none"> <li>• Trigonometry in Automobile Accident Reconstruction</li> <li>• Vector Equations in Three Dimensions</li> <li>• Project: Precalculus with Limits</li> <li>• Project: Tangent Lines to Sine Curves</li> </ul> <p><b>DIFFERENTIATION:</b></p> <p><a href="http://www.sosmath.com/index.html">http://www.sosmath.com/index.html</a>  <a href="http://www.coolmath.com">www.coolmath.com</a>  <a href="http://www.mathleague.com">www.mathleague.com</a>  <a href="http://www.interactmath.com">www.interactmath.com</a>  <a href="http://www.themathpage.com/aTrig/trigonometry.htm">http://www.themathpage.com/aTrig/trigonometry.htm</a></p> <p><b>RESOURCES:</b>  Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. al., © 2007</p>